

in the exact point during problem solving at which they switch from one strategy to the other.

Water-jug Problem

A related kind of problem, which also requires a carefully ordered sequence of steps, is the water-jug problem. The water-jug problem was featured in a slightly different form in *Die Hard 3*; try solving it here – without a ticking bomb to worry about!

A mother sends her boy to the river in order to measure out three quarts of water. The mother gives her son a seven-quart bucket and a four-quart bucket. How can the son measure out exactly three quarts of water using nothing but these two buckets and not guessing as to the amount of water that he brings home? Try to solve this problem before reading on.

This is a simple example of a water-jug problem. To solve the problem, the son merely needs to fill the seven-quart bucket and pour the water into the four-quart bucket. He is now left with three quarts of water in the seven-quart bucket.

Consider now a slightly harder water-jug problem:

A circus owner sends one of his clowns to bring back from a nearby river seven gallons of water to give to the elephants. He gives the clown a five-gallon bucket and a three-gallon bucket and tells him to bring back exactly seven gallons of water. How can the clown measure out exactly seven gallons of water using nothing but these two buckets and not guessing at the amount?

This problem is a bit more difficult. First, the clown needs to fill the five-gallon bucket. Next, he must pour the water into the three-gallon bucket. Having done this, he throws the three gallons back into the river. He now takes the two gallons left in the five-gallon bucket and pours them into the three-gallon bucket. By filling the five-gallon bucket again, he will now have five gallons in that bucket and two gallons in the other bucket, for a total of seven gallons.

Of course, there are “water-jug” problems that do not make use of either jugs or water. Such problems, which are identical in form to the water-jug problem but which make use of different entities in the problem statements, are called “problem isomorphs.” Although they are parallel in form to the original problems, research by John Hayes and Herbert Simon (1976), among others, has shown that problem isomorphs are sometimes easier and some times harder than the original problem. In other words, changing the content of a problem can change its difficulty, even if the form of the problem remains unchanged. So consider a problem isomorph for the water-jug problem:

A cook needs one gram of salt to season a special meal he is cooking. When he opens the drawer to get a measuring spoon, he finds out that he has only an eleven-gram measuring spoon and a four-gram measuring spoon. How can the cook measure out exactly one gram of salt using nothing but these two spoons without guessing at the amount?

What the cook needs to do is to fill the four-gram measuring spoon first and pour the salt into the eleven-gram measuring spoon. Then, he needs to repeat this procedure two more times. The third time, he will be able to pour only three of the four grams into the eleven-gram spoon. He will be left with one gram of salt on the four-gram spoon. Now consider a similar problem:

With a five-minute hourglass and a nine-minute hourglass, what is the quickest way to time a thirteen-minute steak?

One strategy for solving this problem is to start both hourglasses and the steak together. After the five-minute hourglass runs out, turn it over. When the nine-minute hourglass runs out, turn over the five-minute hourglass. It will run for four minutes, yielding a total time of thirteen minutes.

The type of problem characterized in this section can be made somewhat more difficult by including three rather than two water jugs, hourglasses, or whatever. Consider, for example, this problem:

You have three jugs – A, B, and C. Jug A has a capacity of eight quarts, Jug B has a capacity of five quarts, and Jug C has a capacity of three quarts. Initially Jug A is full, but the two smaller jugs are empty. How can you divide the contents of the largest jug evenly between the largest and middle-sized jugs—that is, between jugs A and B?

This problem is quite a bit harder than the problem that has preceded it. See if you can solve it, and then check the solution (Figure 3–11) at the end of the chapter. As shown in the figure, you pour three quarts from Jug A into jug C, and then pour Jug C into Jug B. Pour three more quarts from Jug A into Jug C. Now pour two quarts from Jug C into Jug B, filling Jug B (five quarts). One quart is left in Jug C. Empty Jug B into Jug A; then pour the one quart from Jug C into Jug B. Fill Jug C again from Jug A. Finally, empty Jug C into Jug B.

The Tower of Hanoi and its Variations

One of the most famous problems in the problem-solving literature is the Tower of Hanoi problem. In this problem, the solver is presented with three sticks and a set of discs mounted on the first of those sticks. The discs are of unequal sizes, and they are mounted so that the largest one is on the bottom, the next largest one is immediately above that, and so on until the top of the pile, which contains the smallest disc. The number of discs varies from one version of the problem to another. The idea is to transfer all of the discs from the first stick to the third stick, using the middle stick for intermediate steps of problem solving. In transferring the discs, the solver is never allowed to place a larger disc on top of a smaller disc. A picture of a typical Tower of Hanoi puzzle is shown in Figure 3–5.

Because this book does not come with a set of discs and sticks, it is necessary to use isomorphs to the Tower of Hanoi problem in order to give you a chance to solve problems of this type. Consider the following isomorph, studied by John Hayes and Herbert Simon (1976):

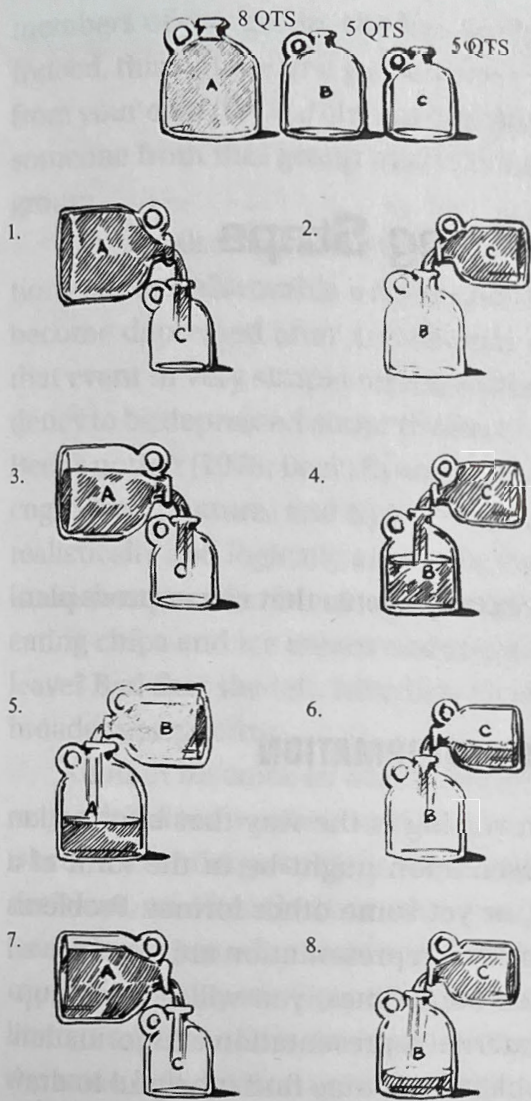


Figure 3-11. Solution to the Water Jug Problem

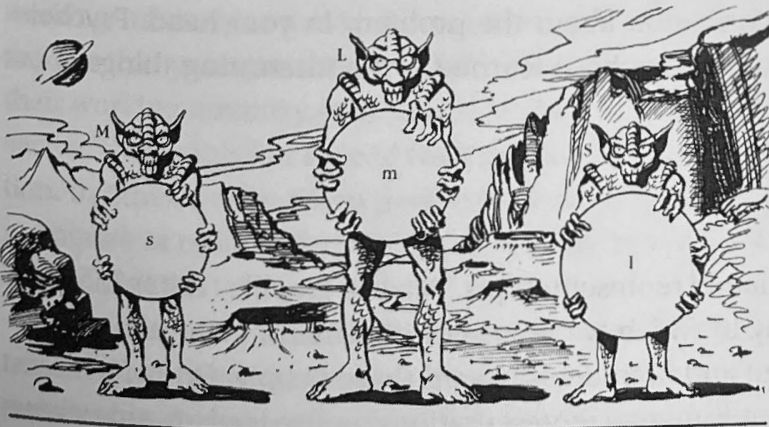


Figure 3-12. Solution to the Extraterrestrial Monsters Problem

1. M	L	S
s	m	l

3. M	L	S
-	l & m	s

5. M	L	S
m	-	l & s

2. M	L	S
s	l & m	-
4. M	L	S
-	m	l & s
6. M	L	S
m	l	s